Potential Flow Interactions with Directional Solidification

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The growth of crystals from melts in terrestrial environments suffers from several natural forms of convection due to buoyancy, volume-change, and thermo-capillary effects, thus giving birth to defects and compositional non-uniformities. Traditionally, this has provided the incentive to explore materials processing under microgravity conditions, in order to study crystal growth in an environment free of buoyancy, with the goal of manufacturing defect-free crystals.

Over the last two decades, a large amount of work has been performed in studying the stability aspects of the solid-liquid interface in a binary solidification process. Most of the earlier studies have investigated the linear behavior of a pure morphological instability,[1,2] while others have addressed nonlinear aspects.[3,4] Furthermore, significant attention has been focused on modifications of the morphological instability under the influence of both natural and externally imposed flows, most of them being viscous in nature.[5,6]

In contrast to these earlier studies, our present investigation focuses on the effects of the potential flow fields typically encountered in Hele-Shaw cells. Such a Hele-Shaw cell can simulate a gravity-free environment in the sense that buoyancy-driven convection is largely suppressed, and hence negligible. Our interest lies both in analyzing the linear stability of the solidification process in the presence of potential flow fields, as well as in performing high-accuracy nonlinear simulations.

In the absence of flow, our linear stability results of the fully time-dependent governing equations for a pure morphological instability (MI) agree closely with the quasi-steady predictions of Mullins and Sekerka.[1] We subsequently impose a uniform parallel flow. The linear stability results indicate that it is possible to stabilize an otherwise unstable interface by imposing a sufficiently strong uniform parallel flow (U). The higher the value of U, the greater the stabilizing effect. This agrees with the experimental work being carried out currently by Zhang and Maxworthy at the University of Southern California. The parallel potential flow gives rise to traveling interfacial waves which move downstream with a velocity on the order of one per cent of the freestream velocity.

As expected, increasing values of the Sekerka number (M) destabilize the interface by increasing the bandwidth of unstable waves. Furthermore, higher values of the surface energy parameter (R) lead to a stabilization of the higher wavenumbers.

Our nonlinear simulations are based on a highly accurate computational approach. The physical domain is analytically mapped into a computational domain. A Fourier spectral method is then employed in the periodic (spanwise) direction, and a sixth order compact finite difference scheme

with spectral-like resolution in the pulling (streamwise) direction. Time advancement is performed by means of a 3rd order Runge-Kutta scheme. This combination has given us results that are accurate to within 1% of the analytically predicted growth rates for the no-flow case at small amplitudes. We have been successful in carrying the simulations to larger amplitudes where nonlinearities start playing important roles. We have followed the interfacial growth to times when the depth of the grooves is comparable to their wavelengths. Usually, at late times the interface is dominated by those wavelengths for which linear theory predicts the largest growth rates.

As a next step, the potential flow is incorporated into our nonlinear simulations. This is performed by using a boundary element technique. This approach can be used to explore a variety of different flow fields, in order to investigate opportunities for suppressing the instability. The boundary element technique is especially advantageous, as it can be combined with various spatial distributions of sources and sinks in the flowfield.

Most of our numerical simulations have been carried on CRAY T90 at the San Diego Super Computing facility at the University of California at San Diego.

References

1. Mullins, W.W., and Sekerka, R.F. *J. of Appl. Phys.*, 35 (444)(1964).

2. Langer, J.S. *Rev. of Mod. Phys.*, 52, 1, 1 (1980).

3. Konstantinos, T., and Brown, R.A. *Phys. Rev. B* 49, 18, 12 724 (1994).

4. Wollkind, D.J., and Segel, L.A. *Phil. Trans. Roy. Soc. London Ser. A*, 268, 351 (1970).

5. Forth, S.A., and Wheeler, A.A. *J. Fluid Mech.*, 202, 339 (1989).

6. Schulze, T.P., and Davis, S.H. *Phys. Fluids* 8 (9), 2319 (1996).